BIPM Capacity Building & Knowledge Transfer Programme

2025 BIPM - TÜBİTAK UME Project Placement

REPORT

Project Name	Development of a Comprehensive Calibration Procedure for Norway's Primary Mass Standards Using the Multiplication and Subdivision Method
Description	This project aims to develop a robust and repeatable calibration procedure for Norwegian mass standards. The focus is on implementing the RealMass software and having a better understanding of the multiplication and subdivision method when different statistical models are used. The goal is to improve the stability and traceability of Justervesenet's primary standards.
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Motivation & Introduction

As the responsible for the Mass Department at Justervesenet, I am closely involved in the calibration and maintenance of the primary mass standards that establish Norway's traceability to the International System of Units (SI). To ensure long-term consistency, transparency, and reproducibility in mass calibrations, it is essential to have a solid and well-documented understanding of the mathematical models, uncertainty evaluations, and physical effects underlying both multiplication/subdivision and substitution weighing methods. This includes a general understanding of the statistical and numerical methods used to process mass calibration data and to evaluate uncertainties, as well as the correct treatment of air buoyancy corrections, centre-of-mass effects, and correlations arising from common input quantities.

In addition, a clearer understanding of mass traceability, its future development following the redefinition of the kilogram, and the role of primary realization methods such as the Kibble balance is required.

Ultimately, the objective is to apply this understanding to the development of an internally maintained and automated calibration software at Justervesenet for primary and secondary mass standards, ensuring robustness, traceability, and long-term sustainability.

Research

1. Determination of the model function

The true mass of the test weight is determined by comparison with a reference standard using a substitution weighing scheme. The measurement model is expressed as:

$$m_t = m_r + \delta m_1 + \delta m_2 + \delta m_3 + \delta m_4 + \delta m_5 + \Delta m_w$$

where m_t is the true mass of the test weight, and the remaining terms represent corrections applied to the reference mass and the weighing process.

1.1 Reference Mass

$$m_r = m + \delta m_r$$

- m_r: True mass of the reference standard at the time of calibration
- m: Nominal or calibrated mass value of the reference standard
- δm_r: Correction given in the calibration certificate of the reference standard

1.2 Drift of the Reference Mass

$$\delta m_1 = \left(\frac{\delta m_r}{1\text{ year}}\right) \cdot \Delta t$$

This term accounts for the **long-term mass drift** of the reference standard between its last calibration and the time of use.

- Δt: Time elapsed since the last calibration (in years)

1.3 Air Buoyancy Correction

$$\delta m_2 = \rho_a (V_t - V_r)$$

This correction accounts for the **buoyancy effect of air**, which depends on the difference in volume between the test weight and the reference standard.

- ρ_a: Air density during weighing
- V_t, V_r: Volumes of the test weight and reference standard, respectively

1.4 Center of Mass (Gravity Gradient) Correction

$$\delta m_3 = (m - \rho_a V_r)\,\delta g\,(z_r - z_t)$$

This term corrects for the **vertical gravity gradient** when the centers of mass of the test and reference weights are at different heights.

- δg: Vertical gravity gradient
- z_r, z_t: Heights of the centers of mass of the reference and test weights

1.5 Volumetric (Thermal) Expansion Correction

$$\delta m_4 = \rho_a (t - t_0)(\alpha_t V_t - \alpha_r V_r)$$

This correction accounts for **changes in volume due to temperature deviations** from the reference temperature.

- t: Ambient temperature during weighing
- t_0: Reference temperature (usually 20 °C)
- α_t, α_r: Coefficients of thermal expansion of the test and reference weights

1.6 Digital Rounding Error

$$\delta m_5 = 0$$

The digital rounding error of the weighing instrument is considered negligible and is therefore set to zero in the measurement model.

This is justified because the mass differences are obtained from substitution weighing cycles (ABBA/ABA), where the balance indication is used only in terms of differences between successive readings. In such weighing schemes, any rounding error due to the finite resolution of the instrument is systematic and symmetric, and therefore cancels out when calculating the mean weighing difference.

1.7 Weighing Difference

$$\Delta m_w = \frac{1}{n}\sum_{i=1}^{n} \Delta\, m_{w_i}$$

This term represents the **mean of the mass differences** obtained from repeated substitution weighings (e.g. ABBA or ABA cycles).

- n: Number of weighing cycles
- Δm_{w_i}: Individual weighing differences

2. Conventional mass

In legal metrology, **conventional mass** is used instead of true mass. It is defined as the result of weighing in air under **conventional reference conditions** (OIML D 28):

- reference temperature: 20 °C
- air density: ρ_0 = 1.2 kg m^{-3}
- reference weight density: ρ_c = 8000 kg m^{-3}

The conventional mass m_c of a weight is the mass of a reference weight (density ρ_c) that balances it in air of density ρ_0. If the true mass m and density ρ of a weight are known, its conventional mass is calculated as:

$$m_c = m\frac{1 - \rho_0/\rho}{1 - \rho_0/\rho_c} = m\frac{1 - (1.2\,\text{kg m}^{-3}/\rho)}{0.99985}$$

Thus, the conventional mass depends only on the mass and density of the weight. For stainless steel weights (ρ ≈ 8000 kg m⁻³), the difference between m_c and m is very small. For other materials, the relative deviation ranges from about -3.6×10^{-4} (silicon) to $+9.4 \times 10^{-5}$ (platinum).

The concept of conventional mass simplifies mass comparisons by avoiding explicit buoyancy corrections during routine weighing. Weighing instruments are commonly adjusted using the conventional mass of adjustment weights. Under standard air conditions, a weighing instrument indicates the conventional mass, making objects of equal mass but different densities comparable.

3. Uncertainty

3.1 Type A Uncertainty

The average weighing difference Δm_w is determined from n ABBA weighing cycles. The corresponding Type A standard uncertainty is:

$$u(\Delta m_w) = \frac{s(\Delta m_w)}{\sqrt{n}}$$

where $s(\Delta m_w)$is the empirical standard deviation of the measured differences. For reliable estimates in high-precision mass determinations, **at least six ABBA cycles** are recommended.

3.2 Type B Uncertainties

Type B uncertainties are evaluated from sources other than statistical analysis:

Reference mass (u_R):

$$u_R = \sqrt{\left(\frac{U_R}{k}\right)^2 + u_{\text{inst}}(m_R)^2}$$

where U_R is the expanded uncertainty from the calibration certificate (coverage factor k, typically 2) and $u_{\text{inst}}(m_R)$ is the uncertainty due to the reference mass instability, estimated from drift history or empirical data.

Air buoyancy correction (u_b):

$$u_b^2 = (V_T - V_R)^2 u_{\rho_a}^2 + \rho_a^2 (u_{V_T}^2 - u_{V_R}^2)$$

where V_T, V_R are the volumes of the test and reference masses, and $u_{\rho_a}, u_{V_T}, u_{V_R}$ are their standard uncertainties.

The reason for the negative sign of the term $\rho_a^2\left(u_{V_T}^2 - u_{V_R}^2\right)$ is that usually the volume of a mass standard is determined only once, so that the air buoyancy corrections of consecutive comparisons are correlated.

Balance sensitivity (u_s):

$$u_s^2 = (\Delta m_w^2)\frac{u^2(m_s)}{m_s^2} + \frac{u^2(\Delta I_s)}{\Delta I_s^2}$$

where m_s is the sensitivity weight and ΔI_s the corresponding change in balance indication.

Balance resolution (u_d):
For digital balances, the limited resolution contributes:

$$u_d = \frac{d/2}{\sqrt{3}}\sqrt{2}$$

where d is the scale interval.

Position effect (u_p):
Differences in readings due to weight positions are corrected by weighing in **interchanged positions**:

$$u_p = \frac{|\, \Delta m_{w1} - \Delta m_{w2} \,|}{2}$$

Balance linearity and repeatability: Minor contributions from the balance's inherent linearity and short-term fluctuations are also included.

4. Correlations

In mass determination, several input quantities are not statistically independent. A typical example is the air density used for buoyancy correction, which is derived from temperature, pressure, and humidity measured with the same instruments across different weighings. As a result, air density values obtained in different measurements are correlated.

According to the GUM, correlation arises when two input quantities depend on one or more common influencing quantities (JCGM 100:2008, F.1.2.3). If two estimates x_1 and x_2 depend on the same underlying quantity q_l, the uncertainty associated with q_l contributes to their covariance. In practice, an instrument reading can be separated into a common (systematic) component, mainly related to calibration uncertainty (Type B), and a random component due to repeatability (Type A). Only the common component contributes to covariance.

Following GUM Equation (F.2), the covariance between two estimates is given by

$$u(x_1, x_2) = \sum_l \frac{\partial F}{\partial q_l}\frac{\partial G}{\partial q_l} u^2(q_l),$$

where the sum includes only quantities common to both estimates.

The correlation coefficient is then

$$r(x_1, x_2) = \frac{u(x_1, x_2)}{u(x_1)u(x_2)}.$$

In mass calibration, correlations between air density values generally have a limited impact on the combined uncertainty, except in cases where buoyancy correction is significant, such as comparisons between materials with very different densities (e.g. stainless steel and platinum-iridium).

5. Realisation of a Mass Scale

Precision mass measurements are typically performed using differential weighings of weights with the same nominal value. When only a single reference mass is available for a set of unknown weights, all unknowns are linked to that reference and determined together as a set. This approach, common in National Metrology Institutes, enables submultiples and multiples of the kilogram to be traced back to a primary standard, such as the platinum–iridium prototype kilogram.

5.1 Weighing Schemes

Mass calibration is carried out using predefined weighing schemes that combine reference and unknown weights. According to OIML R 111, weights are organized in decades following 1–2–5 × 10^n subdivisions. Each scheme produces multiple mass differences, leading to an over-determined system that is solved by least-squares adjustment, enabling both determination of unknown masses and detection of weighing errors. The calibration is propagated from one decade to the next by using previously calibrated masses as references, ensuring consistency across the entire mass scale.

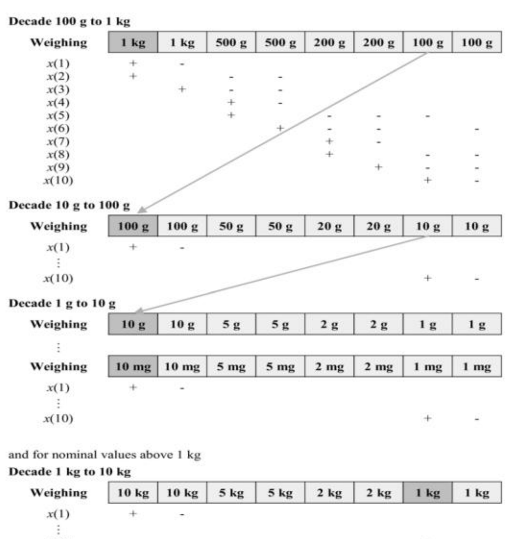


Figure 1. Example of the realization of a mass scale in the range from 1 mg to 10 kg according to a weighing scheme with seven unknown mass standards and ten weighing operations in each decade (Borys *et al* 2008).

5.2 Least-Squares Adjustment Methods

Different mathematical methods can be applied to solve the system of equations, depending on the number of constraints considered:

- **Lagrange multiplier method**: Provides a solution for the unknown masses. The resulting variance-covariance matrix contains only type A uncertainties.
- **Gauss–Markoff approach**: Adjusts the values of both unknowns and references and provides a variance-covariance matrix including both type A and type B uncertainties.

When only one constraint is used, both methods yield the same solution. The least-squares adjustment also produces the variance-covariance matrix, giving standard uncertainties and covariances for the weights, which are essential for high-accuracy mass determinations.

5.3 Practical Considerations

1. **Subdivision of nominal values**: Decades are typically subdivided into 1, 2, 2, 5 combinations to allow flexible weighing.
2. **Auxiliary standards**: Sufficient auxiliary weights must be available to complete the set.
3. **Identification**: Reference, auxiliary, and unknown weights must be clearly marked.
4. **Over-determined scheme**: More weighing equations than unknowns allow internal error control.
5. **Computer-aided evaluation**: Matrix calculations simplify least-squares adjustments and uncertainty analysis.

This procedure ensures that the mass scale is traceable, internally consistent, and optimized for minimum uncertainty, providing a robust link from the primary standard down to the smallest calibrated weights.

6. Kibble Balance

The Kibble balance, formerly known as the Watt balance, is a precision instrument used to realize the SI unit of mass (kilogram) in terms of fundamental constants. Unlike traditional mass standards, which rely on physical artifacts, the Kibble balance links mass to electrical measurements, ultimately traceable to the Planck constant h.

The Kibble balance operates in two modes:

6.1 Velocity Mode (Moving Coil)

When a coil moves vertically through a magnetic field B, it generates an induced voltage U according to Faraday's law:

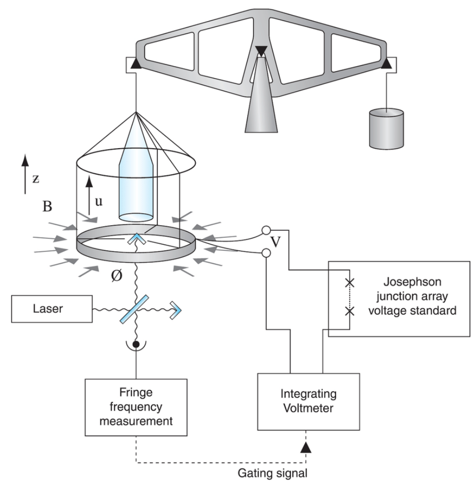
$$U = BLv$$

Where:

- U= induced voltage (V)
- B= magnetic flux density (T)
- L= length of wire in the magnetic field (m)

- v= coil velocity (m/s)

This allows the magnetic flux factor BL to be measured accurately.



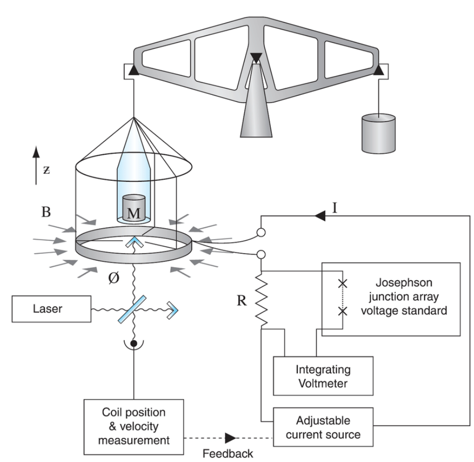
6.2 Force Mode (Static Equilibrium)

The same coil is used to balance the gravitational force on a mass m with an electromagnetic force:

$$mg = BLI$$

Where:

- m= mass (kg)
- g= local gravitational acceleration (m/s²)
- I= current in the coil (A)



6.3 Combining the Two Modes

From velocity mode: $BL = \frac{U}{v}$. Substituting into force mode:

$$mg = I\frac{U}{v} \implies m = \frac{UI}{gv}$$

This is the fundamental equation of the Kibble balance, showing that mass is determined by electrical measurements traceable to physical constants.

6.4 Importance in Metrology

1. **Traceable to fundamental constants:** Mass is linked to constants of nature, eliminating reliance on physical artifacts.
2. **High precision:** Uncertainties can be as low as 10^{-8}.
3. **Universality:** Measurements can be reproduced anywhere with quantum standards.
4. **Stability over time:** Mass realizations are immune to drift or contamination of physical prototypes.
5. **Supports SI redefinition:** Provides the practical means to implement the kilogram definition based on constants of nature.

7. Other research activities

- The determination of air density was reviewed using the CIPM-2007 formula, and air density values calculated from measured temperature, pressure, and humidity were compared with those obtained at the TÜBİTAK UME Mass Laboratory.

- The determination of the volume of mass standards was also reviewed, including a practical example in the volume laboratory followed by the corresponding calculations.

Conclusions and Future Work

This project strengthened the technical understanding of primary mass calibration using the multiplication and subdivision method, including the measurement model, uncertainty evaluation, and the correct treatment of correlations. It also provided a clearer view of mass traceability following the redefinition of the kilogram and the role of primary realization methods such as the Kibble balance. The knowledge gained supports the consistent application of least-squares adjustment methods and the correct use of tools such as the RealMass software. Future work will focus on applying this understanding in routine calibrations at Justervesenet and on the gradual development of an internally maintained and automated calibration software for primary and secondary mass standards, ensuring robust results, clear traceability, and long-term sustainability.

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